G.I. Barenblatt's Contributions to Fracture Mechanics

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In 1959, Barenblatt (PMM 23, no. 3, p. 434) published a famous paper that laid the foundation of the cohesive crack model and thus revolutionized fracture mechanics. Building on Zheltov and Khristianovich's 1955 observation (Izvestia ANS Otd. Tekh. Nauk no. 11) that the opposite faces of the crack must have a smooth closing at the crack tip (i.e., become asymptotically parallel at the crack tip), so as to prevent infinite strain, Barenblatt in 1959 introduced two crucial hypotheses which still underlie nonlinear fracture mechanics today: 1) the requirement of smooth closing implies that, near the tip, cohesive stresses of a certain magnitude must be transmitted between the crack faces, and 2) the cohesive stress is a function of the crack opening width, defined by a law that is a material property. Using Sneddon's (Fourier Transforms, 1951 book) axisymmetric solution for the distribution of opening displacements along the radius of a circular (penny-shaped) crack in an infinite isotropic elastic solid, Barenblatt calculated the magnitude of cohesive stresses required to counteract the load and achieve the required smooth closing. This is obviously equivalent to requiring that the stress intensity factors (or energy release rates with respect to crack length) due to load and to the cohesive stresses must cancel each other (which is the way the requirement is stated in subsequent literature).

To make an analytical solution feasible, Barenblatt in 1959 further introduced the simplifying hypothesis that, near the crack tip, the cohesive stress distribution along the crack can also be treated as a material property. Later, once computer analysis of crack propagation became feasible, a unique softening law relating the cohesive stress to the cross-crack relative displacement became the standard hypothesis, which is, for a small crack in a large structure, equivalent to Barenblatt's 1959 hypothesis. In 1964, Barenblatt (PMM 28, no.4, p. 630) extended his model to include micro-defects or microcracking; in 1966 to kinetics of quasistatic crack growth and long-time

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strength (with Entov and Salganik, in Russian in Mekhanika Tverdogo Tela no. 5, p. 53, no. 6, p. 40). The formulation of the cohesive crack model was completed in 1968 by Rice (in Fracture, Liebowitz, ed., Acad. Press, Vol. 2) who showed, via his path-independent J-integral (published in the same volume), that the flux of energy into the crack tip, representing the energy release rate (with respect to crack length) from an elastic structure, is equal to the work of cohesive stress on the crack opening displacement during crack propagation. This work, equal to the area under the curve of the softening law, represents the fracture energy—a basic material constant which was introduced in 1921 by Griffith (Phil. T. Roy. Soc.A, 221, p. 163) and was shown in 1958 by Irwin (in Fracture, Flügge, ed., Springer, p. 551) to be uniquely related to the critical stress intensity factor, or fracture toughness.

Barenblatt's 1959 paper in PMM was published in Russian. Despite its prompt translation into English in the U.S., it was only in 1962 that his follow-up, more detailed, paper in English (Adv. in Appl. Mech. 7, p. 55), gained a world-wide attention. Meanwhile, Dugdale in 1960 (JMPS 8, p. 100) published an apparently similar paper dealing with cracks (or, in his words, slits) that have a large plastic zone in front. The uniformly distributed stresses of plastic yielding ahead of the crack tip have a similar effect on crack closing as the nonuniform cohesive stresses. They also eliminate the singularity of stress and strain at the crack tip, cause the total stress intensity factor due to load to vanish and thus ensure smooth crack-tip closing. Most subsequent authors, however, did not cite Barenblatt's original 1959 paper in Russian, being unaware of its English translation. In their superficial comments on the cohesive crack model origin, they cited only Dugdale's 1960 and Barenblatt's second (1962) paper, in English. This gave the erroneous impression that Barenblatt was not the first.

However, even if Barenblatt's 1959 paper in Russian did not exist, his 1962 paper alone sufficed to make him the creator of the cohesive crack model. Perfunctory readers did not realize that Dugdale's 1960 paper dealt with a different problem—the plastic deformation near the tip of a sharp slit, and not the propagation of a cohesive crack with progressive softening. His plastic yielding zone could extend indefinitely without any actual crack growth. Dugdale's model included neither softening nor a fracture criterion and, in fact, was called the "strip yield model" (see, e.g., Tada, Paris, Irwin handbook, 1985). It is interesting to note, though, that if a cut-off is added to Dugdale's model so that the material would suddenly break at a certain critical relative displacement, the model would become equivalent to a special limiting case of cohesive crack model in which the stress-displacement function is rectangular, terminating with a sudden stress drop. While such a function is not realistic, it has the advantage of allowing instructive analytical solutions in some interesting cases (which is why it was repeatedly used in Bažant and Planas's 1998 textbook Fracture and Size Effect, CRC Press).

A rigorous comparison to Griffith's (1921) original model of brittle fracture was presented in 1967 by Willis (JMPS 15, p. 151), who also showed (*ibid.* p. 157) the correct extension of Barenblatt's concept to dynamic crack propagation. In 1976, Hillerborg et al. (Cem. Concr. Res. 6, p. 773), using the name"fictitious crack model", proposed that a cohesive (or fictitious) crack can initiate in concrete without any pre-existing crack or notch wherever the tensile strength limit is reached (this recognized the fact that, unlike metals, the concept of crack nucleation is meaningless for concrete since the material is full of densely spaced cracks to begin with, at all scales from nano to macro). Subsequently, Petersson (Dissertation, Lund Inst Tech. 1981), using this version of cohesive crack model, developed an effective fracture simulation algorithm for concrete structures in which the crack and its fracture process zone are not small compared to the structure size. In this context, it was also shown that equally good, and often superior, simulations of fracture in concrete or geotechnical structures can be obtained with the crack band model (Bažant and Oh, Mat. & Struct. 16, p. 155), which is, for vanishing width, asymptotically equivalent to Barenblatt's cohesive crack model. In many cases of large structures, the crack band gives nearly the same energy release rate, while it can directly capture the finiteness of the fracture process zone width, whose main role is to limit parallel crack spacing. Thanks to using a tensorial damage constitutive model, the crack band has the advantage of being able to reproduce the effect of high crack-parallel compression on the apparent opening-mode fracture energy, which can be quite significant in quasibrittle materials such as concrete, rock and fiber composites (H. Nguyen et al., PNAS June 2020, and JAM-ASME, July 2020).

Barenblatt's works on fracture mechanics are distinguished by clarity, logic of reasoning and accuracy, and are devoid of unnecessary mathematical formalism. He returned to fracture mechanics repeatedly and provided a number of innovative inspirational ideas and solutions, whose hallmark is the scaling laws. In particular, the following deserves to be pointed out: • In his early work in mid 1950s, he clarified some mechanics problems in the rock burst of deep mining stopes, and in the hydraulic fracturing of oil-bearing strata (Barenblatt and Khristianovich 1955, Izvestia ANS, No. 11.; Barenblatt 1956, PMM 20, No. 3). • Barenblatt formulated useful scaling laws for fatigue crack growth. In his insightful study of subcritical fatigue crack growth with Botvina in 1981 (Fatigue of Eng. Mat. & Str. 3, p. 193), he demonstrated an application of his idea of incomplete similarity (Barenblatt, ibid., 1979) in the linear cyclic growth range. He and Botvina showed that, in metals, the Paris law exponent varies linearly with the square root of the ratio of structure size to the plastic zone size, the latter being proportional to Irwin's characteristic length. They supported this conclusion by analysis of previously published data. Interestingly, this conclusion is similar to what was observed in tests of fatigue fracture growth in concrete or rock (Bažant & Xu 1991, ACI Mat. J. 88 (4) p. 390; Kirane & Bažant, IJ Fatigue 70, 2014, p. 93; ibid. 83, 2016, 209–220; Mech. Res. Com 2015, 68, p. 60), except that the size effect on the exponent was not detected, probably due to insufficient size range and higher scatter in concrete testing. • Aside from various applications in fluid mechanics, Barenblatt's concept of intermediate asymptotics (Similarity, self-similarity and intermediate asymptotics, Consultants Bureau 1979) has been invoked in Bažant's 2005 book (Scaling of Structural Strength, Elsevier 2005) and paper (PNAS 2004, p. 13400) to support the Types I and II size effect laws of quasibrittle fracture. • Barenblatt pointed out already in his 1962 paper that, in an atomic lattice on the nanoscale, cracks must propagate in discrete jumps, as a series of instabilities, later called snap-through instabilities. Consequently, the stress-displacement relation of a cohesive crack may deviate, even on the nanoscale, from the equilibrium path and must be irreversible. • Recently, with Monteiro and Rycroft (PNAS 2012), his collaborators in Berkeley, Barenblatt focused again on nanoscale fracture growth. Returning to his previous idea of nanoscale dynamic snap-through jumps in crack length, he showed that, aside from fracture energy G_f and material tensile strength f_t, the nanoscale fracture in an atomic lattice must depend on a third material property, the mass density ρ, through a nanoscale characteristic length $\lambda = [h(\rho E)^{-1/2}]^{1/4}$ (where E is the nanoscale Young's modulus, and h is Planck's constant).

What must have helped Barenblatt to come up with transformative advances was that the Russian school of mechanics, to which he belonged, was world-class in the 1950s. His 1959 transformational paper spurred great technological progress. His contributions to fracture mechanics contributed to safety, efficiency and durability of engineering structures. Cohesive fracture mechanics became important for both metallic and quasibrittle structures.

Applications to the latter include: 1) geomechanics, particularly hydraulic fracturing for oil or gas extraction; 2) coarse-grain or toughened ceramics; 3) polymers and polymer-fiber composites (particularly the design of modern composite airframes); 4) concrete structures (which has finally been recognized by American Concrete Institute whose 2019 design code is the first one based on fracture mechanics, underlying its 2019 design specifications for the size effect in shear strength of beams and slabs); 5) floating sea ice plates (whose load capacity and forces applied by moving ice on the ocean platforms must consider the size effect of cohesive fracture); 6) fracture of bones and other biomaterials (in which cohesive fracture generates size effect); 7) fracture of brittle materials (even metallic thin films) on the micrometer scale at which they become quasibrittle and exhibit deterministic size effect; etc. However, it must be noted that, in the light of recent results (H. Nguyen et al., 2020, ibid.), for quasibrittle materials the cohesive crack model requires a significant correction when there is a significant crack-parallel stress, in-plane or out-of-plane, in order to take into account the effect of a wide fracture process zone.

Commenting on Barenblatt's impact, it is timely to mention also an environmental connotation. Cracking and its localization in concrete, whose correct prediction must take into account cohesive stress effects, governs the ingress of moisture with corrosive agents into concrete structures, which undermines durability and thus has enormous environmental consequences. It is known (though widely ignored) that, despite the great recent progress in reducing the cement content of modern concretes, the worldwide production of cement and concrete is on the verge of greatly exceeding the CO_2 emissions from all the cars and trucks in the world. One way to mitigate this problem in the long run would be to double the lifetimes of concrete structures and pavements. This goal would, of course, depend partly on realistic fracture assessments, which are facilitated by Barenblatt's ideas.